[9] Cheng, P., "Dynamics of a Radiating Gas with Application to Flow over a Wavy Wall," *AIAA Journal*, Vol. 4, No. 2, Feb. 1966, pp. 238–245.

[10] Traugott, S., "A Differential Approximation for Radiative Transfer with Application to Normal Shock Structure," Research Report 34, Dec. 1962, Martin Co.

[11] Van Dyke, M., *Perturbation Methods in Fluid Mechanics*, Academic Press, New York, 1964.

[12] Pritulo, M., "On the Determination of Uniformly Accurate Solutions of Differential Equations by the Method of Perturbation of Coordinates," *Prikladnaza Matamatika e Mechanika*, Tom XXVI, No. 3, 1962, pp. 444–448.

JANUARY 1970 AIAA JOURNAL VOL. 8, NO. 1

Temperature Distribution and Effectiveness of a Two-Dimensional Radiating and Convecting Circular Fin

S. SIKKA* AND M. IQBAL†

University of British Columbia, Vancouver, Canada

An analysis is made of the heat-transfer characteristics of a circular fin dissipating heat from its surface by convection and radiation. The temperature is assumed uniform along the base of the fin and constant physical and surface properties are assumed. There is radiant interaction between the fin and its base. Two separate situations are considered. In the first situation heat transfer from the end of the fin is neglected. Solution of the linear conduction equation with nonlinear boundary conditions has been obtained by a least-squares fit method, and also by the finite difference method and the results compared. Results are presented for a wide range of environmental conditions and physical and surface properties of the fin. In the second situation, heat transfer from the end of the fin is also included in the analysis. The solution for the second situation is obtained by a finite-difference procedure only. It is shown that neglecting heat transfer from the end is a good approximation for long fins or for fins of high thermal conductivity material.

Nomenclature

a = radius of circular fin, ft
A = $\epsilon_1 \sigma a T_0^3/k$, radiation-conduction parameter, dimensionless
F = configuration factor, dimensionless
h = heat-transfer coefficient, Btu/hr ft^2 °R
k = thermal conductivity of fin, Btu/hr ft °R
l = fin length, ft
L = l/a, dimensionless fin length
N = ha/k, convection parameter, (Biot number), dimensionless
Q = rate of heat loss from fin, Btu/hr
r,z = cylindrical coordinates for fin, ft
R,Z = r/a, z/a, dimensionless cylindrical coordinates
r_2 = fin base radius, ft
T = absolute temperature, °R
T_0 = fin base temperature, °R
T_∞ = fluid bulk temperature, °R
T^* = effective radiation environment temperature, °R
α = coefficient of absorptivity, dimensionless
ϵ = coefficient of emissivity, dimensionless
σ = Stefan-Boltzmann constant, 0.1714×10^{-8} Btu/hr ft^2 °R^4
η = fin effectiveness
β = r_2/a, dimensionless
λ = $(T - T_\theta)/T_0$, dimensionless temperature at any point in the fin
λ_∞ = T_∞/T_0, dimensionless fluid bulk temperature
λ^* = T^*/T_0, dimensionless effective radiation environment temperature

Subscripts

1 = fin surface
2 = base surface
∞ = fluid bulk

Superscripts

* = effective radiation environment

Received December 30, 1968; revision received July 25, 1969. Use of the Computing Centre facilities at the University of British Columbia and the financial support of the National Research Council of Canada are gratefully acknowledged. Thanks are due to B. D. Aggarwala, Department of Mathematics, University of Calgary, and K. Teng of the University of British Columbia Computing Centre for useful suggestions.

* Graduate Student.

† Associate Professor, Department of Mechanical Engineering.

Introduction

THE solar energy absorbed by a space vehicle by direct incidence or reflection from planets and the energy generated by electronic instruments in the vehicle itself have to be dissipated away to the surroundings. To limit the large amounts of heat-transfer area required on the space vehicle, fins are extensively used.

Although several studies have been made on the steady-state heat transfer for radiating fins, most of the work has been for the one-dimensional model. Shouman[1] has obtained an exact general solution for a constant cross-sectional area fin. However, he considered a one-dimensional model with no fin-to-base interaction. Liu[2] had earlier developed an exact solution for the rectangular profile fin. Lieblein[3] obtained a finite difference solution for the rectangular fin while Bartas and Sellars[4] determined the fin effectiveness for one-dimensional heat flow in rectangular fins by numerical methods. Sparrow and Eckert[5] considered the effects of mutual irradiation occurring between a fin and its adjoining base surfaces. Sparrow, Eckert, and Irvine[6] used numerical iterative methods to analyze the effectiveness of plane radiating fins with mutual irradiation. Chambers and Somers[7] numerically determined the fin efficiency for a flat annular fin. Sparrow, Miller, and Jonsson[8] used finite difference methods to calculate the fin effectiveness for one-dimensional heat flow in annular fins with mutual irradiation between the black radiator elements. Recently, Sparrow and Nie-

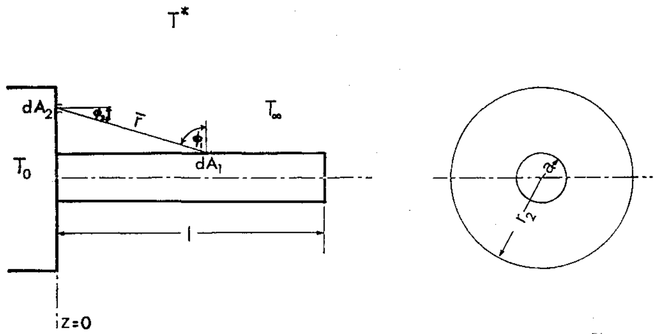

Fig. 1 Circular fin geometry.

werth[9] have given the numerical and linearized solutions for the one-dimensional heat conduction in convecting-radiating fins.

Apparently the literature contains very few studies for the two-dimensional heat flow and they have all used finite-difference techniques. Holstead and Holdredge[10] have used finite-difference techniques to solve the problem of trapezoidal profile fins with no base interaction for both one-dimensional and two-dimensional cases. Sparrow, Jonsson, and Minkowycz[11] have obtained the solution for the two-dimensional heat flow in fin-tube radiators by a finite-difference procedure.

In the present report, a series solution whose coefficients are determined by the least squares fit method, has been employed to analyze the two dimensional heat flow in a circular fin. Heat is dissipated by radiation and convection from its surface. In addition, radiation interchange with the base is included. The results of the least squares fit method have been compared with a finite difference solution. The finite difference procedure is described in the Appendix.

Formulation and Solution of the Problem

Consider a circular fin of radius a and length l as shown in Fig. 1, subject to the following conditions: 1) the temperature is uniform along the base of the fin, 2) there is no incident radiation on the fin so that there is rotational symmetry, 3) the physical and surface properties of the fin and base materials are invariant with temperature, 4) the fin is of isotropic homogeneous material, and 5) the fin has radiation interaction with the base but multiple interactions with the base are not considered.

Two cases are considered and they are dealt with separately in two sections. Section A deals with the case in which heat transfer from the end of the fin is neglected in comparison with that from its sides. In Sec. B, the heat transfer from the end is also included and the results obtained are compared with those of Sec. A.

A. No Heat Transfer from the End of the Fin

The steady state differential equation of heat conduction and the boundary conditions can be written as follows. Energy equation:

$$(\partial^2 T/\partial r^2) + (1/r)\cdot(\partial T/\partial r) + (\partial^2 T/\partial z^2) = 0 \qquad (1)$$

Boundary conditions:

$$1)\ \text{at } z = 0,\ T = T_0 \qquad (2)$$

$$2)\ \text{at } z = l,\ \partial T/\partial z = 0 \qquad (3)$$

$$3)\ \text{at } r = 0,\ \partial T/\partial r = 0 \qquad (4)$$

4) The fourth boundary condition at $r = a$ involves the energy balance of an elemental area dA_1.

Assuming gray body properties for the fin, (i.e., $\alpha_1 = \epsilon_1$), the energy balance equation reduces to

$$-k\cdot dA_1\cdot\left.\frac{\partial T}{\partial r}\right|_{r=a} = dA_1\cdot(1 - F_{dA_1\to A_2})\cdot\epsilon_1\sigma T_{r=a}{}^4 - \epsilon_1\sigma T^{*4}\cdot dA_1\cdot(1 - F_{dA_1\to A_2}) + dA_1\cdot h(T_{r=a} - T_\infty) - dA_1\cdot F_{dA_1\to A_2}\cdot\epsilon_2\sigma T_0{}^4\cdot\epsilon_1 + dA_1\cdot F_{dA_1\to A_2}\cdot\epsilon_1\sigma T_{r=a}{}^4$$

where the configuration factor $F_{dA_1\to A_2}$ introduced above denotes the fraction of the total energy emitted by dA_1 that is intercepted by A_2. Rearrangement of the terms yields

$$\left.\frac{\partial T}{\partial r}\right|_{r=a} = -\frac{\epsilon_1\sigma}{k}\cdot T_{r=a}{}^4 + \frac{\epsilon_1\sigma}{k}\cdot T^{*4}\cdot(1 - F_{dA_1\to A_2}) - \frac{h}{k}\cdot(T_{r=a} - T_\infty) + \frac{\epsilon_1\epsilon_2\sigma}{k}\cdot T_0{}^4 F_{dA_1\to A_2} \qquad (5)$$

The governing equation for the temperature distribution in the fin and the boundary conditions may be rephrased into convenient dimensionless forms by introducing dimensionless variables as follows:

$$\lambda = (T - T_0)/T_0,\ \lambda_\infty = T_\infty/T_0,\ \lambda^* = T^*/T_0$$

$$R = r/a,\ L = l/a,\ Z = z/a$$

In terms of these new variables, Eq. (1) and the boundary conditions [Eqs. (2–5)] become

$$(\partial^2\lambda/\partial R^2) + (1/R)\cdot(\partial\lambda/\partial R) + (\partial^2\lambda/\partial Z^2) = 0 \qquad (6)$$

Boundary conditions:

$$1)\ \lambda|_{z=0} = 0 \qquad (7)$$

$$2)\ \left.\frac{\partial\lambda}{\partial Z}\right|_{Z=L} = 0 \qquad (8)$$

$$3)\ \left.\frac{\partial\lambda}{\partial R}\right|_{R=0} = 0 \qquad (9)$$

$$4)\ \left.\frac{\partial\lambda}{\partial R}\right|_{R=1} = -A(1 + \lambda_{R=1})^4 + A\lambda^{*4}(1 - F_{dA_1\to A_2}) - N(1 + \lambda_{R=1}) + N\lambda_\infty + A\epsilon_2 F_{dA_1\to A_2} \qquad (10)$$

where $A = \epsilon_1\sigma a T_0{}^3/k$ is a radiation-conduction parameter and $N = ha/k$ is actually the Biot number but here is referred to as the convection parameter since it is being used to study the effect of the convective heat-transfer coefficient h.

The solution of the energy equation (6) satisfying the three boundary conditions (7–9) is as follows:

$$\lambda = \sum_{n=0}^{\infty}\left(C_n\cdot\left\{I_0\left[\frac{(2n+1)\pi}{2L}\cdot R\right]\right\}\cdot\sin\left[\frac{(2n+1)\pi}{2L}\cdot Z\right]\right) \qquad (11)$$

The unknown coefficients $C_n(n = 0, 1, 2, \ldots, \infty)$ can be determined by point matching or least squares fitting at a finite number of points on the boundary, choosing equally spaced points along the length of the fin and the matching or fitting to Eq. (10). The point matching method involves developing a polynomial with a number of unknown coefficients which exactly satisfies the differential equation. The boundary condition is then exactly satisfied only at a number of points on the surface. The number of points are the same as the number of unknown coefficients in the polynomial. The resulting set of simultaneous equations is then solved for the unknown coefficients. However, we have employed the least-squares fit method since it is known to be more efficient than the point matching one.[13] This method also involves developing a polynomial which exactly satisfies the differential equation but it utilizes more points than the unknown coefficients of the polynomial. The boundary conditions at these points are then not exactly satisfied but it ensures a better fit to the boundary as a whole. From boundary-condition Eq. (10), the expression to be minimized is obtained as

$$S = \Sigma_1 \left\{ \frac{\partial \lambda}{\partial R}\bigg|_{R=1} - [-A(1 + \lambda_{R=1})^4 + A\lambda^{*4}\cdot (1 - F_{dA_1\to A_2}) - N(1 + \lambda_{R=1}) + N\lambda_\infty + A\epsilon_2 F_{dA_1\to A_2}] \right\}^2$$

where Σ_1 denotes summation over all points in the region $0 \leqslant Z \leqslant L$.

Using the series solution for λ as expressed in (11), the previous expression takes the following form:

$$S = \Sigma_1 \left(\sum_{n=0}^{\infty} \left\{ C_n \cdot \frac{(2n+1)\pi}{2L} \cdot I_1\left[\frac{(2n+1)\pi}{2L}\right] \times \sin\left[\frac{(2n+1)}{2L}\cdot Z\right] \right\} + A \left\{ 1 + \sum_{n=0}^{\infty} \left(C_n \cdot I_0 \left[\frac{(2n+1)\pi}{2L}\right] \times \sin\left[\frac{(2n+1)}{2L}\cdot Z\right] \right) \right\}^4 + N \left\{ 1 + \sum_{n=0}^{\infty} \left(C_n \cdot I_0 \left[\frac{(2n+1)\pi}{2L}\right] \times \sin\left[\frac{(2n+1)\pi}{2L}\cdot Z\right] \right) \right\} - A\lambda^{*4}(1 - F_{dA_1\to A_2}) - N\lambda_\infty - A\epsilon_2 F_{dA_1\to A_2} \right)^2 \quad (12)$$

Since the function is nonlinear in the unknown parameters C_n, it is first locally linearized by Taylor's expansion and then the least-squares fit criterion applied, i.e., $\partial S/\partial C_n = 0$, $(n = 0, 1, 2, \ldots, \infty)$. Using only m terms of the series, we get a system of linear algebraic equations which are solved to give the coefficients C_n, $(n = 0, 1, 2, \ldots, m - 1)$. The process is then iterated upon until

$$\left| \frac{C_n^{\text{new}} - C_n^{\text{old}}}{C_n^{\text{old}}} \right| < 10^{-7}$$

In this report, 7 coefficients and 100 points on the boundary were chosen. The least-squares fit method is also described in Refs. 13 and 14.

Thus, for known values of λ_∞, λ^*, A, N, L, and ϵ_2 the temperature distribution can be determined if the configuration factor is known. The configuration factor was evaluated on the lines of the method described in Ref. 12 and the expression is given below.

$$F_{dA_1\to A_2} = \frac{1}{\pi}\cdot \tan^{-1}\left[\frac{(\beta^2 - 1)^{1/2}}{Z}\right] + \frac{Z}{\pi}\left\{ \frac{1}{2}\cdot \cos^{-1}\left(\frac{1}{\beta}\right) - \frac{\phi}{(\phi^2 - 4\beta^2)^{1/2}} \cdot \tan^{-1}\left[\left(\frac{(\beta - 1)(\phi + 2\beta)}{(\beta + 1)(\phi - 2\beta)}\right)^{1/2}\right] \right\} \quad (13)$$

where $\beta = r_2/a$ and $\phi = Z^2 + \beta^2 + 1$.

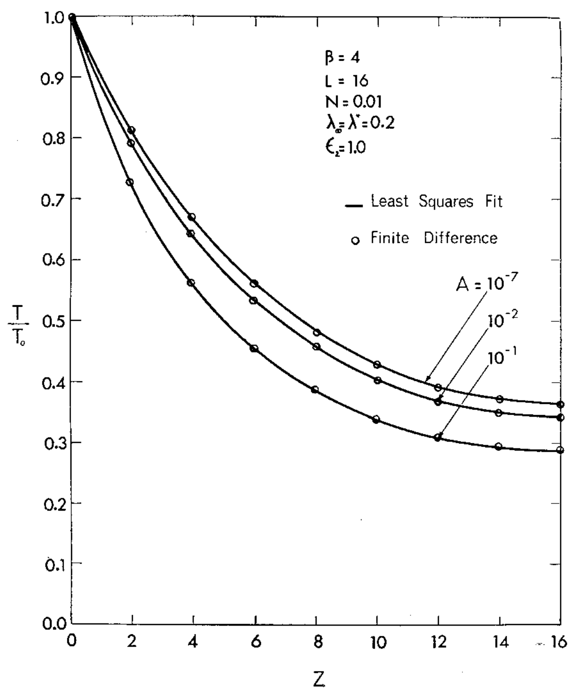

Fig. 2 Effect of radiation-conduction parameter A on the axial temperature distribution.

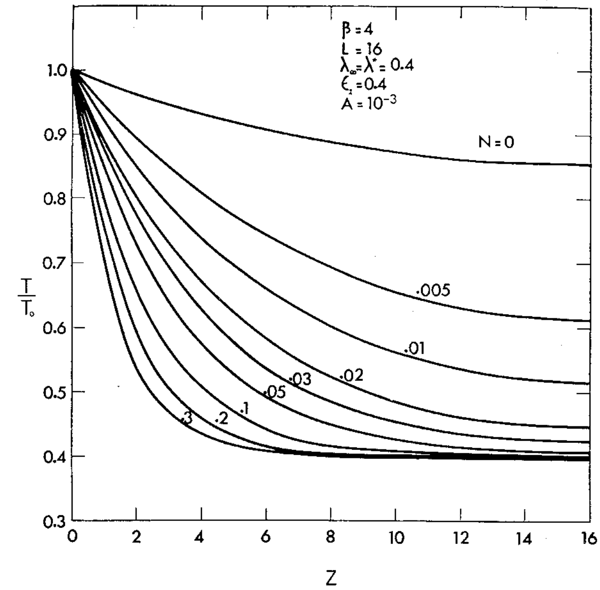

Fig. 3 Effect of convection parameter N on the axial temperature distribution.

Discussion of Results

Fin temperature distributions

The temperature distribution along the axis ($R = 0$) is represented in Fig. 2. It shows that at any point on the axis the temperature decreases with increasing values of the radiation-conduction parameter A. This is as expected since high values of A mean a low value of k or high values of fin radius a or base temperature T_0, each of which decreases the ratio T/T_0. The slope at the end of zero, as it should be, indicating no heat transfer from the end.

Figure 3 shows the effect of the variation of the convection parameter N on the axial temperature distribution. An increase in N causes more heat loss by convection and therefore lower surface and axial temperatures. The surface-temperature distribution is compared with the axial temperature distribution in Fig. 4. The surface temperature is lower than the axial temperature at any section and the difference is larger for fins of low thermal conductivity.

Fin effectiveness

The heat-transfer performance of the fin can be expressed in terms of the fin effectiveness as

$$\eta = \frac{Q}{Q_{\text{ideal}}} = \frac{\int_0^l 2\pi a \left[-k \cdot \frac{\partial T}{\partial r}\bigg|_{r=a} \right] dz}{2\pi a l [\epsilon\sigma(T_0^4 - T^{*4}) + h(T_0 - T_\infty)]}$$

where the ideal heat flux is defined by assuming that the entire fin is at the base temperature and radiates without interference.

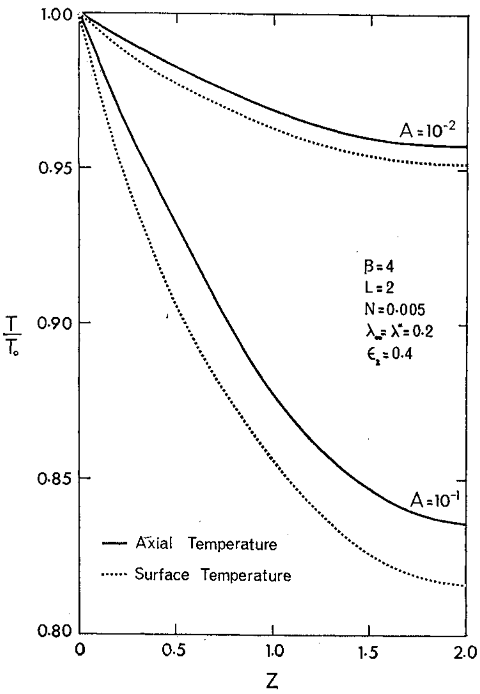

Fig. 4 Comparison of surface and axial temperature distributions.

Nondimensionalizing all the terms, the expression for fin effectiveness becomes

$$\eta = -\frac{1}{L}\left[\frac{\int_0^L \frac{\partial\lambda}{\partial R}\Big|_{R=1} dZ}{A(1-\lambda^{*4}) + N(1-\lambda_\infty)}\right] \tag{14}$$

The effect of the variation of various parameters on the fin effectiveness has been presented. Due to the presence of seven independent parameters only a few representative cases have been presented.

It has been noted that an increase in either of the parameters, the base surface emissivity ϵ_2 or the fin base radius β decreases the fin effectiveness since the greater radiant interaction of the fin with the base suppresses the heat transfer from the fin to the surroundings. Increase of λ_∞ or λ^* also decreases the fin effectiveness in most cases although the effect depends upon the values of other parameters.

Figure 5 shows the effect of the variation of N on η. An increase in the value of N decreases the surface temperature, thus reducing drastically the heat transfer by radiation. This reduction is normally greater than the increase in the convection term, the over-all effect thus being to decrease the fin effectiveness. Increasing values of the fin length reduce the temperature of the fin surface. This decreases the fin effectiveness as illustrated in Fig. 6.

A comparison has been made in Table 1 of the values of fin effectiveness obtained by this work with that by Sparrow and Niewerth.[9] In Table 1, β has been taken as one to avoid bringing in the effect of fin-to-base interaction. The comparison could, however, be made only for a very limited range of values since only a few results were available in Ref. 9 Although the difference in values is small, since high conductivity fins (low values of A) are considered, the fin effectiveness as obtained by this work is lower throughout the range. This is because the two-dimensional model is considered here which causes lower surface temperatures. The greater variation in fin body temperatures causes lower fin effectiveness since it represents greater departure from the isothermal fin case.

Comparison of the least-squares fit results with the finite difference solution

The results of the temperature distribution obtained by the two methods are illustrated in Fig. 3. The values agree with each other very closely. For high values of the radiation-conduction parameter A, there is an agreement up to four significant figures. However, even with this close agreement in temperature distribution, there is an appreciable deviation in the values of fin effectiveness as can be seen from Fig. 6. The deviation depends upon the number of series terms and boundary points. When more series terms and more boundary points are taken in the least-squares fit method, the fin effectiveness values approach those obtained by the finite difference technique. Fin effectiveness values obtained by the least-squares fit method for 7 series terms and 100 boundary points and for 20 series terms and 400 boundary points are compared with those of the finite difference method. Thus, the least-squares fit method results can approach the finite difference values, although at the expense of increased computer time.

B. Heat Transfer from the End of the Fin is Not Neglected

For fins of short length, it may not be very correct to neglect the heat transfer from the end of the fin as compared to that from its sides. This situation is analyzed in this section.

The energy equation and the other three boundary conditions remain unaltered. Only boundary condition 2) [Eq. (3)] is altered. The boundary condition at the end of the fin now takes the following form:

$$-k\cdot\frac{\partial T}{\partial z}\Big|_{z=l} = \epsilon_1\sigma(T_{z=l}^4 - T^{*4}) + h(T_{z=l} - T_\infty) \tag{15}$$

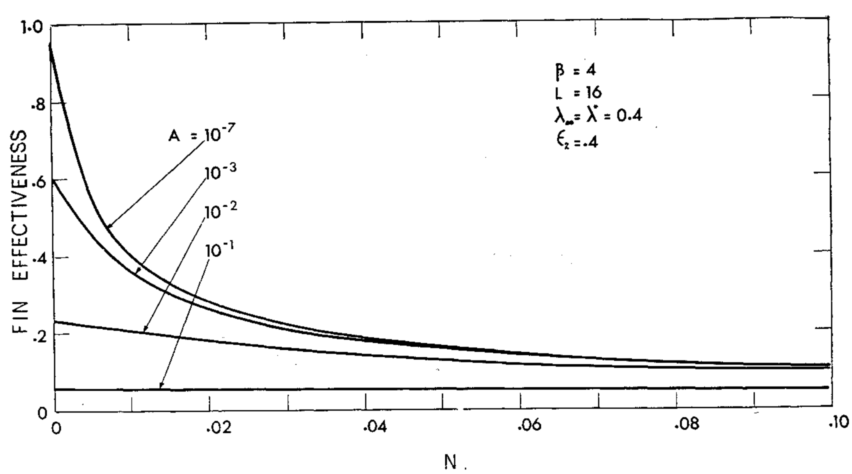

Fig. 5 Effect of N on the fin effectiveness.

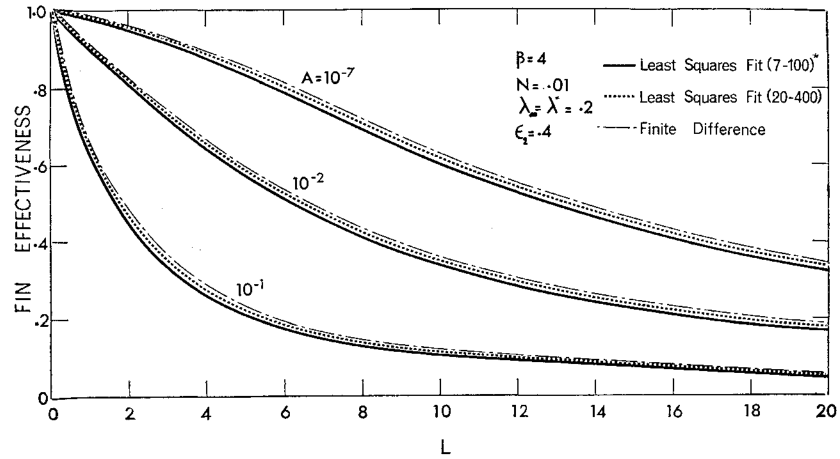

Fig. 6 Effect of fin length on the fin effectiveness. (* denotes 7 terms in the series and 100 boundary points.)

Employing the dimensionless parameters described earlier in Sec. A, the above equation is rendered dimensionless and it then takes the form

$$\left.\frac{\partial \lambda}{\partial Z}\right|_{Z=L} = -A(1 + \lambda_{Z=L})^4 + A\lambda^{*4} - N(1 + \lambda_{Z=L}) + N\lambda_\infty \tag{16}$$

The finite difference procedure was then applied to solve the system of Eqs. (6, 7, 10, 9, 16). The procedure used was similar to the one used in Sec. A. The results obtained were compared with the finite difference results of Sec. A (no heat transfer from the end) and plotted in Figs. 7 and 8.

The fin effectiveness for this condition is re-evaluated as follows:

$$\eta = \frac{Q}{Q_{\text{ideal}}} = \frac{\int_0^l 2\pi a \left[-k\cdot \left.\frac{\partial T}{\partial r}\right|_{r=a}\right] dz + \int_0^a 2\pi r \left[-k\cdot \left.\frac{\partial T}{\partial z}\right|_{z=l}\right] dr}{(2\pi a l + \pi a^2)[\epsilon\sigma(T_0^4 - T^{*4}) + h(T_0 - T_\infty)]} \tag{17}$$

On nondimensionalization the preceding expression takes the following form:

$$\eta = -\left(\frac{2}{2L+1}\right)\cdot\left[\frac{\int_0^L \left.\frac{\partial \lambda}{\partial R}\right|_{R=1} dZ + \int_0^1 R \left.\frac{\partial \lambda}{\partial Z}\right|_{Z=L} dR}{A(1 - \lambda^{*4}) + N(1 - \lambda_\infty)}\right] \tag{18}$$

Comparison of Section A with Section B

Figure 7 illustrates the results of axial temperature distribution obtained by the two boundary conditions stated in Secs. A and B. For low values of A (i.e., high thermal conductivity), agreement between the two conditions is reached at comparatively smaller values of L. This is more clearly demonstrated by the plots for fin effectiveness in Fig 8. Both high values of L or low values of A reduce the difference in the results obtained by the two conditions. However, even for very high thermal conductivity of the fin the marked difference in fin effectiveness, especially at low values of L, will still remain. This is due to the inherent difference in definitions of fin effectiveness for the two cases (Eqs. 14 and 18). Even if heat transfer from the end is negligible and the term

$$\int_0^1 \left.\frac{\partial \lambda}{\partial Z}\right|_{Z=L} R dR$$

is neglected, the difference in the factors $2/(2L + 1)$ and $1/L$ will cause a marked difference in the values of fin effectiveness, especially for low values of L.

Conclusions

The problem of two-dimensional heat flow in a circular fin having radiant interaction with the fin base has been analyzed. The solution is obtained by a series expansion and the least-squares fit method. Results are presented in terms of axial temperature distribution and fin effectiveness. The problem is also solved by a finite difference solution and the results obtained by the two methods are compared. For materials with high thermal conductivity, the results agree closely with the one-dimensional model.

The investigation reported herein has also considered the effect of heat transfer from the end, an effect that has generally been neglected in prior studies of radiating fins. This problem has been solved by a finite difference procedure. The results demonstrate that the effect of heat transfer from the end of the fin becomes small for fin materials of high thermal conductivity or for long fins.

Appendix: Finite Difference Approximation

The elliptic partial differential equation (6) was solved using a square grid. Using the standard 5-point approxima-

Table 1 Comparison of fin effectiveness values with those obtained by Sparrow and Niewerth[9] ($\beta = 1$, $\epsilon_2 = 0.4$)

Values of this work			Reduced to values of Ref. 9					
A	L	N	$N_r = 2AL^2$	$N_{cv} = 2NL^2$	$\lambda_\infty = \theta_\infty$	$\lambda^* = \theta^*$	$\eta_{\text{this work}}$	η, Ref. 9
10^{-2}	3.873	0.005	0.3	0.15	0.7	0.7	0.691	0.728
10^{-2}	3.873	0.005	0.3	0.15	0.9	0.9	0.681	0.718
10^{-3}	12.25	0.005	0.3	1.5	0.7	0.7	0.552	0.580
10^{-3}	12.25	0.005	0.3	1.5	0.9	0.9	0.541	0.571
10^{-2}	5.477	0.01	0.6	0.6	0.7	0.7	0.539	0.577
10^{-2}	5.477	0.01	0.6	0.6	0.9	0.9	0.522	0.558
10^{-3}	17.32	0.006	0.6	3.6	0.7	0.7	0.391	0.420
10^{-3}	17.32	0.006	0.6	3.6	0.9	0.9	0.380	0.409

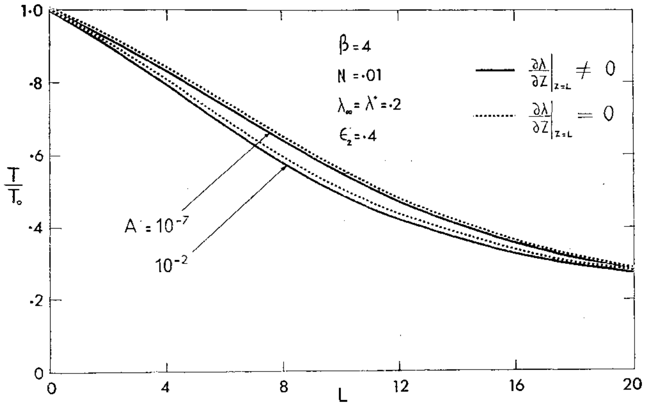

Fig. 7 Comparison of axial temperature distribution obtained for two conditions.

tion of the Laplacian, the finite difference scheme for the interior points is

$$\lambda_{i,j} = \frac{1}{2[(1/a_1^2) + (1/b^2)]}\cdot\left[\lambda_{i,j+1}\left(\frac{1}{b^2} + \frac{1}{2bR}\right) + \lambda_{i,j-1}\left(\frac{1}{b^2} - \frac{1}{2bR}\right) + \frac{1}{a_1^2}(\lambda_{i+1,j} + \lambda_{i-1,j})\right]$$

where i, j and a_1, b are the subscripts and step sizes for the Z and R directions, respectively (Fig. 9a).

On the boundaries, however, we do not try to satisfy the differential equation but satisfy the boundary conditions only. Using the forward-difference approximation for the derivative of a function at a point, one obtains, in general, for a function ϕ at any boundary $\phi_n = \frac{2}{3}[2\phi_{n-1} - \frac{1}{2}\phi_{n-2} + h\cdot(\partial\phi_n/\partial n)]$, e.g., for the boundary $C - D$ ($R = 1$) for a 80×10 grid,

$$\lambda_{i,11} = \tfrac{2}{3}\{2\lambda_{i,10} - \tfrac{1}{2}\lambda_{i,9} + b[-A(1 + \lambda_{i,11})^4 + A\lambda^{*4}(1 - F_{dA_1\to A_2}) - N(1 + \lambda_{i,11}) + N\lambda_\infty + A\epsilon_2 F_{dA_1\to A_2}]\}$$

A coarse grid was first used to obtain a rough shape which was used as the initial distribution for a finer grid. A number of values of the over-relaxation parameter were tried and although the optimum value varied with the variable parameters A, N, etc. of the problem, a value of 1.90 seemed to be most suitable.

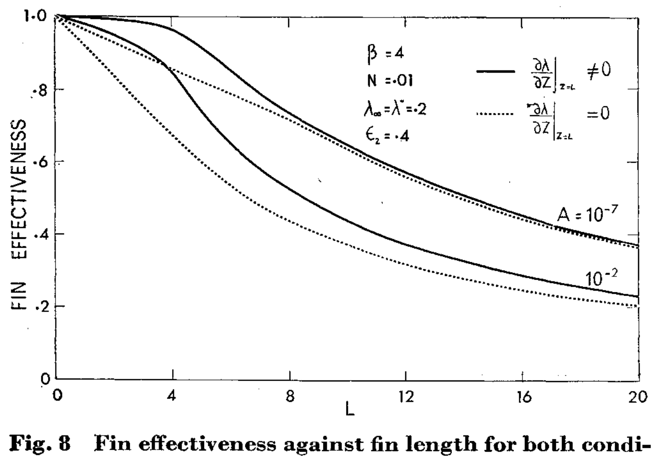

Fig. 8 Fin effectiveness against fin length for both conditions.

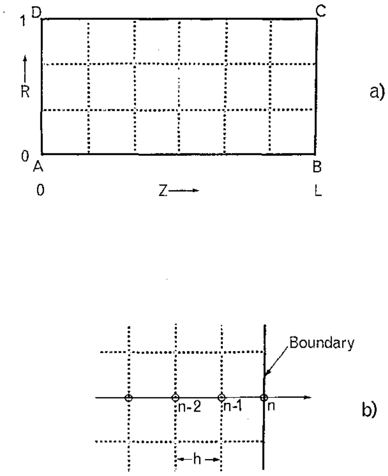

Fig. 9 Finite difference representation.

References

[1] Shouman, A. R., "An Exact General Solution for the Temperature Distribution and the Radiational Heat Transfer Along a Constant Cross-Sectional-Area Fin," Paper 67-WA/HT-27, 1967, American Society of Mechanical Engineers.

[2] Liu, C., "On Minimum-Weight Rectangular Radiating Fins," *Journal of the Aerospace Sciences*, Vol. 27, 1960, pp. 871–872.

[3] Lieblein, S., "Analysis of Temperature Distribution and Radiant Heat Transfer Along a Rectangular Fin of Constant Thickness," TN D-196, 1959, NASA.

[4] Bartas, J. G. and Sellers, W. H., "Radiation Fin Effectiveness," *Transactions of the ASME, Series C: Journal of Heat Transfer*, Vol. 84, 1960, pp. 73–75.

[5] Sparrow, E. M. and Eckert, E. R. G. "Radiant Interaction Between Fin and Base Surfaces," *Transactions of the ASME, Series C: Journal of Heat Transfer*, Vol. 84, 1962, pp. 12–18.

[6] Sparrow, E. M., Eckert, E. R. G., and Irvine, T. F., Jr., "The Effectiveness of Radiating Fins with Mutual Irradiation," *Journal of the Aerospace Sciences*, Vol. 28, No. 10, 1961, pp. 763–772.

[7] Chambers, R. L. and Somers, E. V., "Radiation Fin Efficiency for One Dimensional Heat Flow in a Circular Fin," *Transactions of the ASME, Series C: Journal of Heat Transfer*, Vol. 81, 1959, pp. 327–329.

[8] Sparrow, E. M., Miller, G. B., and Jonsson, V. K., "Radiating Effectiveness of Annular Finned Space Radiators, Including Mutual Irradiation Between Radiator Elements," *Journal of the Aerospace Sciences*, Vol. 29, No. 11, 1962, pp. 1291–1299.

[9] Sparrow, E. M. and Niewerth, E. R., "Radiating, Convecting and Conducting Fins: Numerical and Linearized Solutions," *International Journal of Heat and Mass Transfer*, Vol. 11, 1968, pp. 377–379.

[10] Holstead, R. D. and Holdredge, E. S., "Radiation Heat Transfer for Straight Fins of Trapezoidal Profile," Paper 67-HT-73, 1967, American Society of Mechanical Engineers.

[11] Sparrow, E. M., Jonsson, V. K., and Minkowycz, W. J., "Heat Transfer from Fin-Tube Radiators Including Longitudinal Heat Conduction and Radiant Interchange Between Longitudinally Nonisothermal Finite Surfaces," TN D-2077, 1963, NACA.

[12] Sparrow, E. M., "A New and Simpler Formulation for Radiative Angle Factors," *Transactions of the ASME, Series C: Journal of Heat Transfer*, Vol. 85, 1963, pp. 81–88.

[13] Sikka, S., Iqbal, M., and Aggarwala, B. D., "Temperature Distribution and Curvature Produced in Long Solid Cylinders in Space," *Journal of Spacecraft and Rockets*, Vol. 6, No. 8, Aug. 1969, pp. 911–916.

[14] Ratkowsky, D. A. and Epstein, N., "Laminar Flow in Regular Polygonal Shaped Ducts with Circular Centered Cores," *The Canadian Journal of Chemical Engineering*, No. 46, 1968, pp. 22–26.